Delayed PD-type controller design

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***Abstract*— A new class of compensators with a PD-type structure are proposed in this paper. Such systems have a dead time inherent to the derivative filter, which cannot be avoided. Using linear time delayed error equations, conditions to assure stability despite such delay are developed. The filter delay is assumed to be bounded by a known value.**

I. Introduction

The PID controller continues to be a key of the industrial control. It has been shown in (Astrom and Hagglund, 1995) that more than 95% of the control loops in 1995 were of PID type, however most of them were PI control. Bialkowski described in 1993 that a typical paper mill has more than 2000 control loops and that 97% use PI control and only 20% of the control loops were found to work well and decrease process variability (Bialkowski, 1993). Ender observed that 30% of installed process controllers operate in manual, that 20% of the loops used the default parameters set by the controller manufacturer, and that 30% of the loops worked poorly because of the equipment problems in valves and sensors (Ender, 1993). Therefore, these are just some evidences that many controllers are put in manual mode, and among those controllers that are in automatic mode, derivative action is frequently switched off (Astrom and Hagglund, 1995).

This tuning practice is motivated by the lack of measurement of the controlled signal's time-derivative (e.g. mechanical systems usually have measurement of the positions, but not of the velocities. This implies the inclusion of extra sensors, the use of state observers, or other techniques, like the dirty derivative developed in (Berghuis and Nijmeijer, 1993), and the Levant differentiator (Berghuis and Nijmeijer, 1998). These techniques are seriously affected by noise, even if it is low, and the use of a filter is necessary. However, once the time-delay introduced by the filter is not viewed in isolation but as an inseparable part of the PID dynamics, the implementation becomes a four-parameter design and there is a lack of a widely used methodology in industry to tune it (Isaksson and Graebe, 2002), furthermore, the use of a derivative filter induces a phase delay which affects the controller performance and it can even destabilize the overall process.

Despite the aforementioned problems, in (Isaksson and Graebe, 2002) it was shown that PID controllers may give superior results to PI control even for very simple processes and that the use of default values of the derivative filter is not necessarily a good idea. (Maghade and Swati, 2011), on the other hand, showed that almost in every situation derivative action with low pass filter improves the properties of the industrial control systems compared to PI control. As a first step to deal with this problem, the design of PD-type controllers including filtering delay is considered. It is proved that asymptotic regulation is achieved despite the delay magnitude, which directly depends on the filter's order. This convergence is guaranteed through the use of delay-differential error equations. Furthermore, the transient response is studied by numerical simulations, and the effectiveness of the proposed approach is illustrated by a fundamental example: the double integrator.

II. Introductory example

Consider the case of a double-integrator $\ddot{x}(t) = u(t)$, with noise η added to the output measurement, and $\dot{x}_1$ is not available for measurement, that is, only $y(t) = x_1(t) + \eta(t)$ is available.

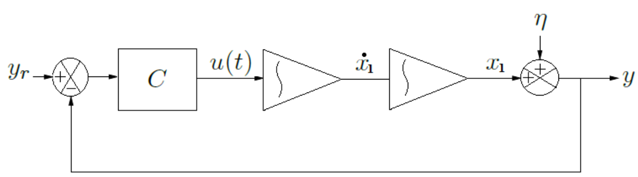

Figure 1. The double-integrator diagram

Applying a standard approach for trajectory tracking, one defines the error signal $e(t) = y_r(t) - y(t)$ and coefficients a_0, a_1 such that $\ddot{e}(t) + a_1\dot{e}(t) + a_0 e(t) = 0$ is stable. This yields

$$(\ddot{y}_r - \ddot{\eta} - u) + a_1(\dot{y}_r - \dot{x}_1 - \dot{\eta}) + a_0(y_r - x_1 - \eta) = 0, \quad (1)$$

and a controller $u(t)$ can be obtained. This control, however, cannot be implemented since it depends on the noise, which is assumed to be unknown.

A natural action is to filter the measured signal y out (assuming high-frequency noise), as depicted in Figure 2.

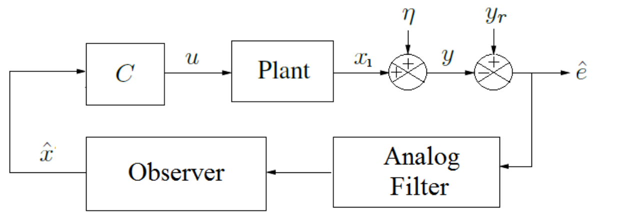

Figure 2. PD control using signal filtering and standard Luenberguer observer.

The drawback of this approach is that a time delay is introduced, so we dispose of $x_1(t-\tau)$ instead of $x_1(t)$ at time t. In this case we define $e(t) = y_r(t) - x_1(t)$, which leads to

$$u(t) = \ddot{y}_r(t) + a_1(\dot{y}_r(t) - \dot{x}_1(t-\tau)) + a_0(y_r(t) - x_1(t-\tau)) \quad (2)$$

and convergence of $x_1(t)$ to $y_r(t)$ is not guaranteed.

III. Proposed approach

In this work, we propose two different schemes to derive the PD controller using delayed information of the error time-derivative, and eventually of the error itself. They are based on delay-differential error equation, whose stability is guaranteed for suitable parameter values. These are now detailed.

A. *Scheme 1: using delayed error time-derivative, and non-delayed error*

Consider the error equation:

$$\ddot{e}(t) + a\dot{e}(t-1) + be(t) = 0 \quad (3)$$

and assume that it is stable for some $a, b \in \Re$. Setting $e(t) = y_r - x_1(t)$ one obtains

$$u(t) = \ddot{y}_r + a(\dot{y}_r(t-1) - \dot{x}_1(t-1)) + b(y_r(t) - x_1(t))$$

Since the time-derivative term is delayed, then it can be computed from the measured input signal, using a filtering scheme that induces a constant time-delay. To this end, and considering a 400hz sampling frequency, it is used

- a low-pass antialias Bessel filter with cutoff frequency of 50hz;
- a finite-impulse-response (FIR) discrete filter (sampling rate of 100hz), whose frequency response is s from 0 to 10hz, and zero from 10 to 50hz This is a combined low-pass and time-differentiator filter design. The frequency response is shown in Figure 3;
- η provided by Simulink's band-limited white noise generator, using noise power of 0.00001 and sampling time of 0.001.

The scheme is depicted in Figure 4 Bessel filter was chosen because they have been optimized to obtain a maximally flat delay response. Altough this is payed by a low selectivity in the frequency response.

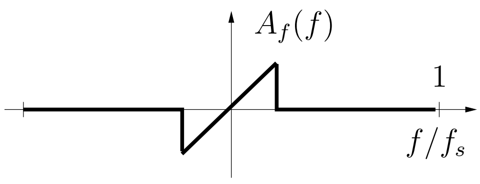

Figure 3. FIR frequency response.

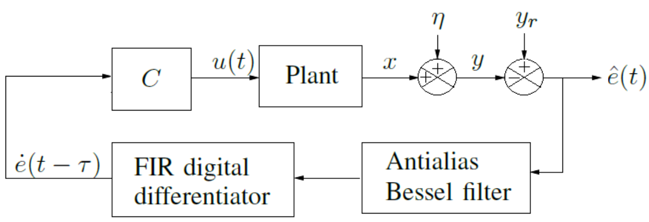

Figure 4. Scheme 1

This is conveyed with a second low-pass filtering stage, included in the FIR differentiator. This double-function FIR design was taken from (Stanley, 1975).

Using this scheme, a simulation was carried out using Matlab/Simulink©R2010a, running on windows 7, 64-bits. The reference signal is

$$y_r(t) = \sin(3t) \quad (4)$$

and the same Bessel filter as in Section II. The FIR filter structure is a direct form, with numerator coefficients [1.3353140743 2.4534859947 3.1983929114 3.3267967565 2.7467372783 1.551489954 0 -1.551489954 -2.7467372783 -3.3267967565 -3.1983929114 -2.4534859947 -1.3353140743], and sample time of 0.01s. The combined filter delay is 0.112s.

Parameters for equation (3), after time scaling, are: $a = 5,\ b = 10$.

Simulation results are shown in Figure 5.

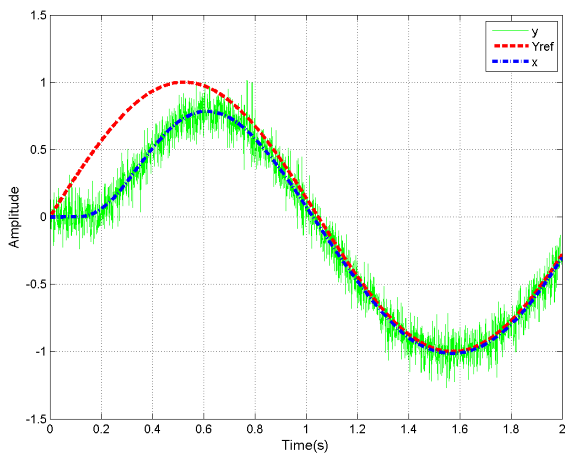

Figure 5. Trajectory tracking, using eq. (3)

The stability criteria for equation (3) are given in the following propositions, taken from (Bellman and Cooke, 1963).

Proposition 1: (Bellman and Cooke, 1963). A system with characteristic function

$$H(s) = s^2 + ae^{-s}s + b \tag{5}$$

is asymptotically stable if and only if:

- for $0 < b < (\frac{\pi}{2})^2$, $0 < a < \frac{\pi}{2} - b(\frac{\pi}{2})^{-1}$;
- for $(\frac{\pi}{2}+2p\pi)^2 < b < \frac{3\pi^2}{4}+4\pi^2(p^2+p)$; $(\frac{\pi}{2}+2p\pi) - b(\frac{\pi}{2}+2p\pi)^{-1} < a < 0$;
- for $\frac{3\pi^2}{4}+4\pi^2(p^2+p) < b < (\frac{\pi}{2}+(2p+1)\pi)^2$;$b(\frac{\pi}{2}+(2p+1)\pi)^{-1} - (\frac{\pi}{2}+(2p+1))\pi < a < 0$;
- for $(\frac{\pi}{2}+(2p+1)\pi)^2 < b < -\frac{\pi^2}{4}+4\pi^2(p+1)^2$; $0 < a < b(\frac{\pi}{2}+(2p+1)\pi)^{-1} - (\frac{\pi}{2}+(2p+1)\pi)$;
- for $-\frac{\pi^2}{4}+4\pi^2(p+1)^2 < b < (\frac{\pi}{2}+(2p+2)\pi)^2$; $0 < a < (\frac{\pi}{2}+(2p+1)\pi) - b(\frac{\pi}{2}+(2p+1)\pi)^{-1}$.

The stability regions are shown in Figure 6, taken from (Brethe, 1994).

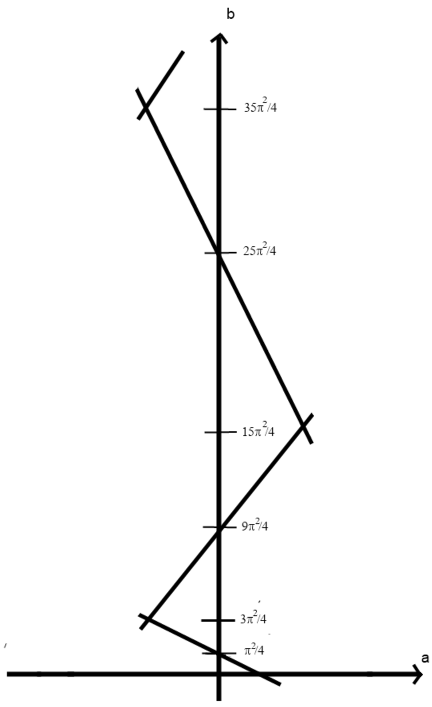

Figure 6. Stability regions for equation (3)

By simulating step responses of equation (3) in the stability region close to the origin, several transient response parameters were measured. They are: the 95% stabilization time (Fig. 7), % overshoot (Fig. 8), and peak time (Fig. 9).

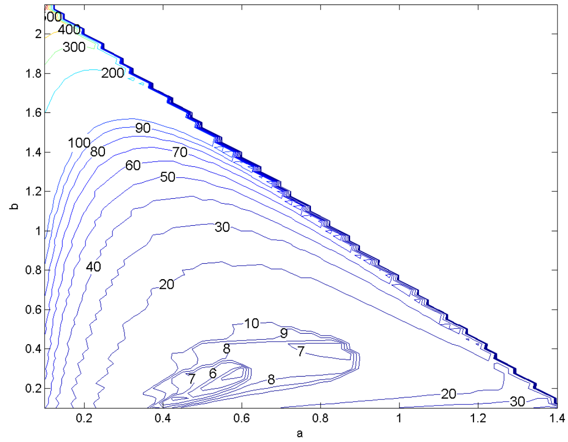

Figure 7. 95% stabilization time, step response of (3).

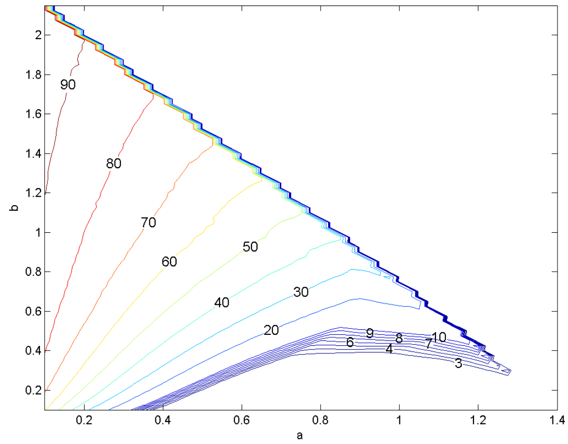

Figure 8. Overshoot (in %), step response of (3).

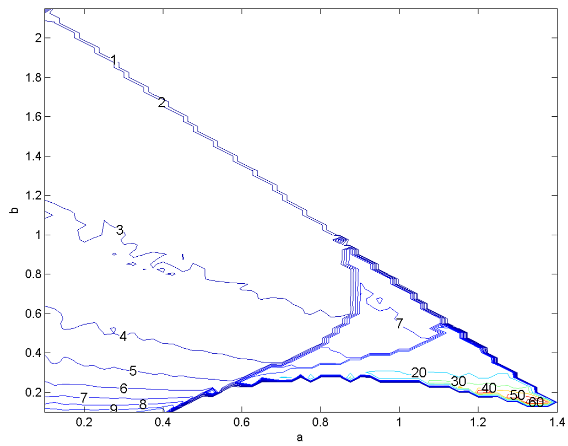

Figure 9. Peak time, step response of (3).

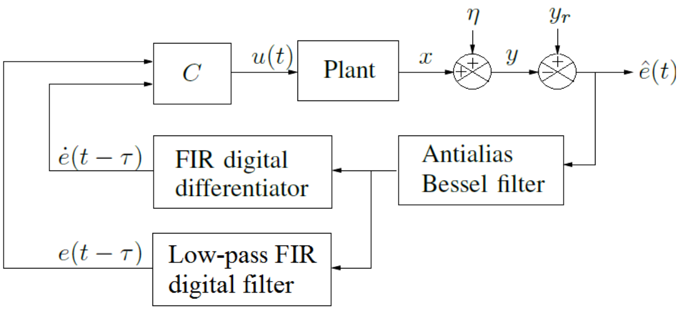

Figure 10. Scheme 2.

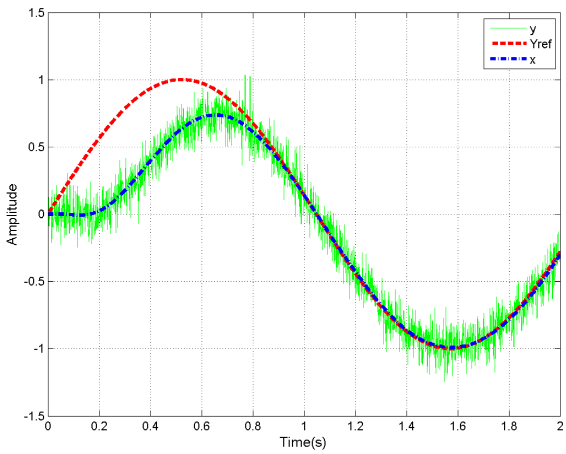

Figure 11. Trajectory tracking, using eq. (6).

B. Scheme 2: using delayed error time-derivative, and delayed error.

Now consider the case where the noise amplitude yields a poor performance with the previous scheme. This requires filtering of the measured signal. In this case, we consider the error equation

$$\ddot{e}(t) + a\dot{e}(t-1) + be(t-1) = 0 \quad (6)$$

which yields

$$u(t) = \ddot{y}_r + a(\dot{y}_r(t-1) - \dot{x}_1(t-1)) + b(y_r(t-1) - x_1(t-1)).$$

This scheme is shown in Figure 10.

In this case, the tracking simulation with the same parameters as in previous example are shown in Figure 11. The stability region is taken from (Brethe, 1994) and shown in Figure 12

The transient step response parameters were also obtained by numerical simulation. They are: the 95% stabilization time (Fig. 13), % overshoot (Fig. 14), and peak time (Fig. 15).

IV. Acknowledgements

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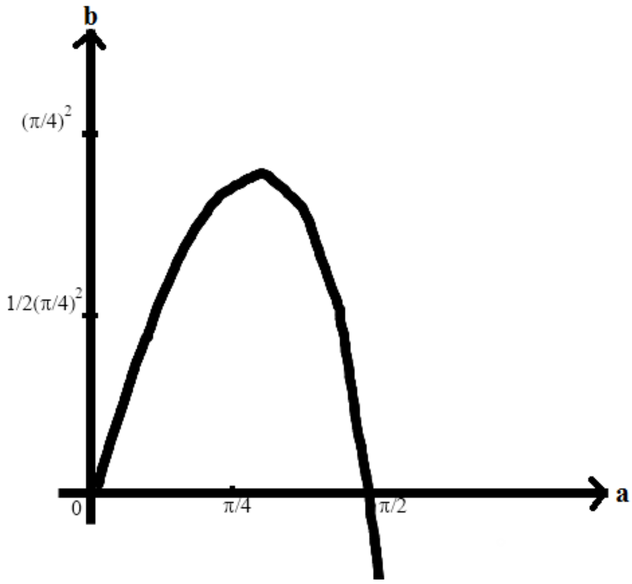

Figure 12. Stability region for eq. (6).

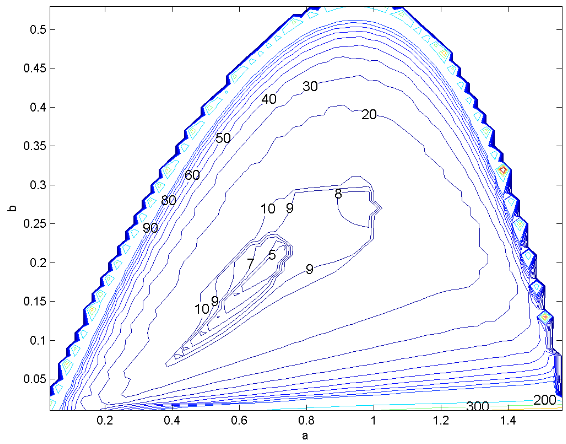

Figure 13. 95% stabilization time, step response of (6).

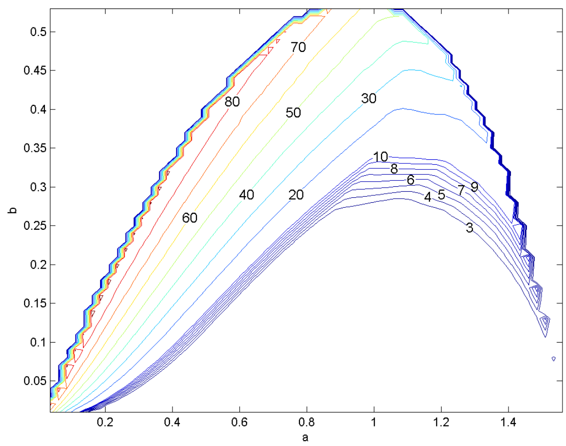

Figure 14. Overshoot (in %), step response of (6).

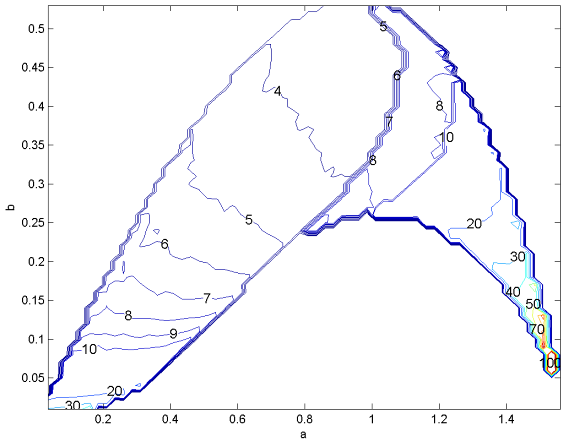

Figure 15. Peak time, step response of (6).

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Conclusions

In this work two different PD-type controllers were proposed. The novelty with respect to the standard PD type, is the inclusion of the delay introduced by the measured signal filtering process. The use of delay-differential error equations allows to guarantee the convergence of the proposed controllers. The use of a simple FIR filter as a combination of a low-pass filter and differentiator displayed a good performance in simulation examples.

The numerical study of transient responses allows to choose suitable controller parameter values. Finally, even if the stability study and transient analysis is done for a unit delay, it can be easily used for any arbitrary delay, by time axis scaling, and any kind of noise can be considered if it does not have important components at the system's operation frequency.

This is a first step towards the general case, where more complex delay-differential error equations are required.

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